Bayesian Conversations

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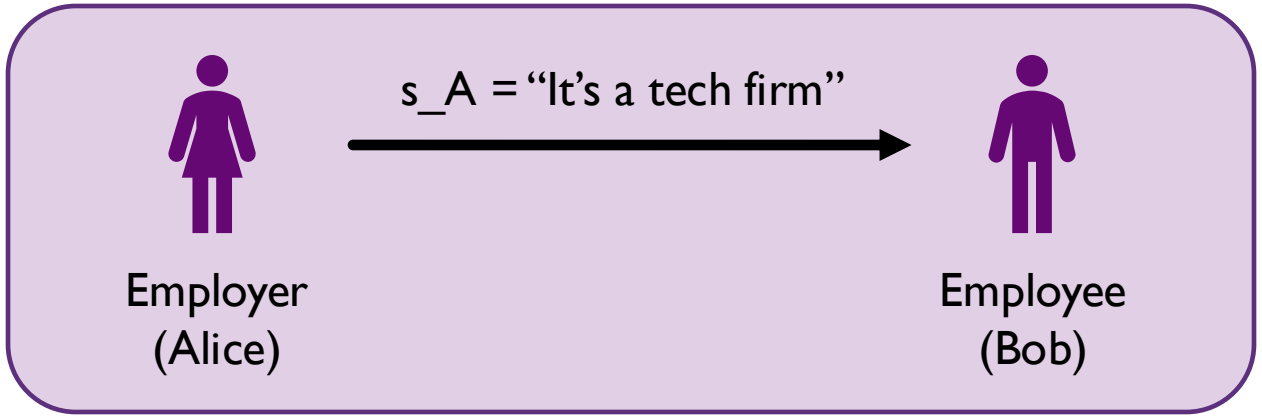
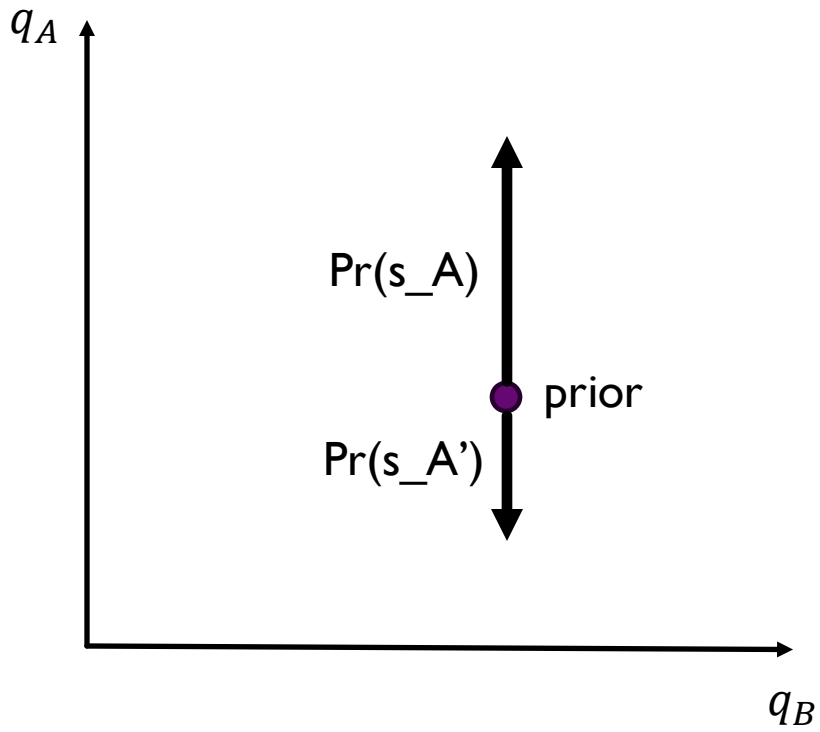
Bayesian Conversations – An Instance

- Hiring scenario
- ***2 players:*** employer and the employee
- ***2 types*** of employer & employee: Programming & Communication
- ***Prior probability distributions*** known: P(θ_A = Prog) and P(θ_B = Prog)
- ***Action taker:*** Employer, decides hire or not
  - Employer wants to find a candidate with the same type as herself
  - Employee simply wants to be hired

Bayesian Conversations

Employer & employee enhance their understanding through sending signals:



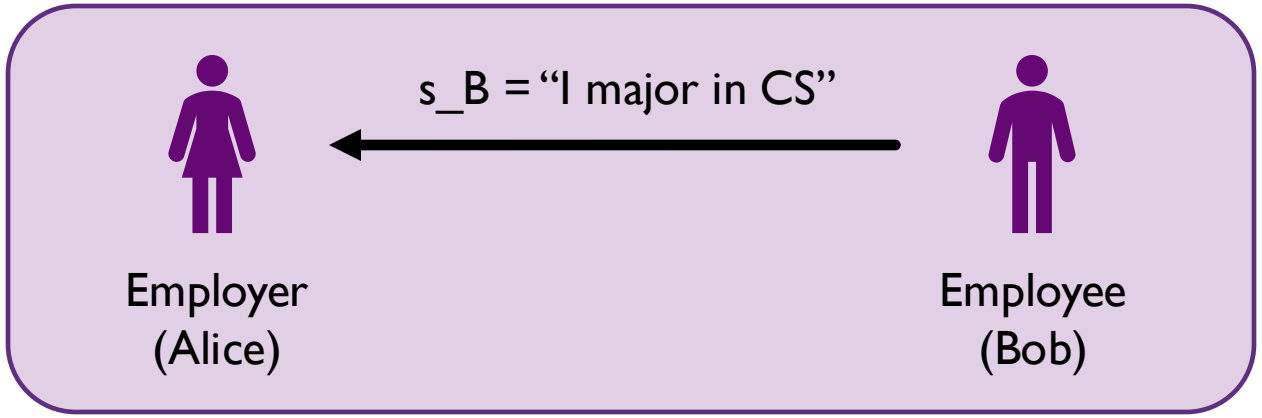


Employee updates his belief q:

$$\Pr(\theta_A = \text{Prog} \mid s_A) = \frac{\Pr(s_A \mid \theta_A = \text{Prog}) \cdot \Pr(\theta_A = \text{Prog})}{\Pr(s_A)}$$

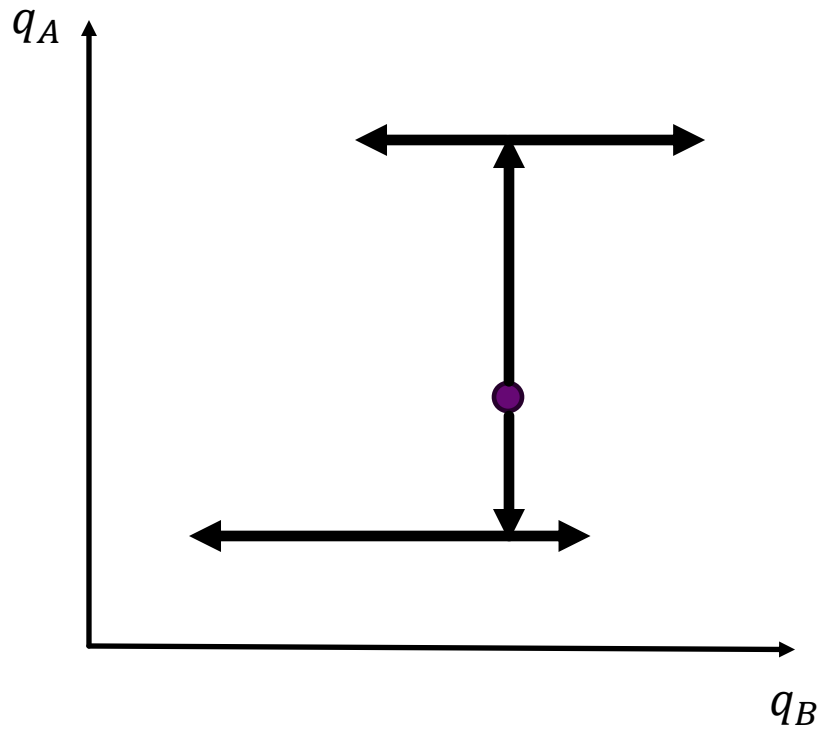
Bayesian Conversations

Employer & employee enhance their understanding through sending signals:



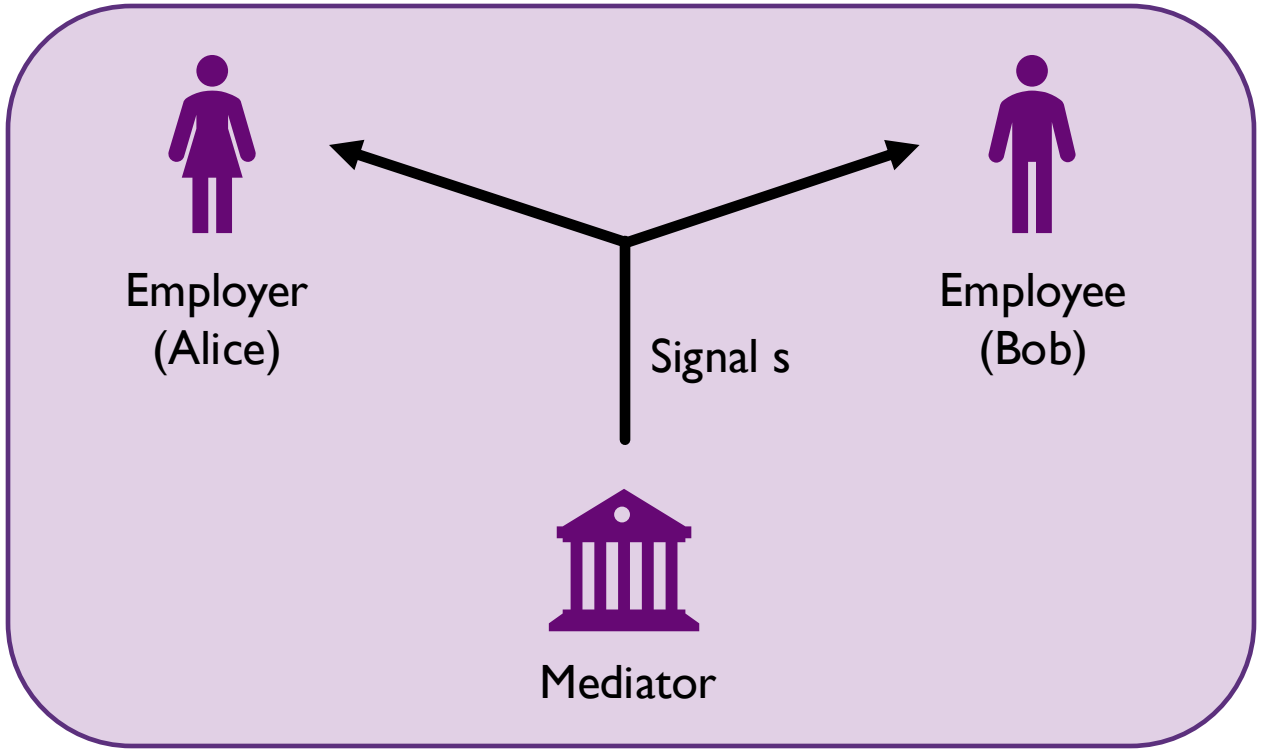
Employer updates his belief q:

$$\Pr(\theta_B = \text{Prog} \mid s_B, s_A) = \frac{\Pr(s_B \mid \theta_B = \text{Prog}, s_A) \cdot \Pr(\theta_B = \text{Prog} \mid s_A)}{\Pr(s_B \mid s_A)}$$



Mediator Protocol

We also introduce communication that involves a trusted-third party ***"mediator".***

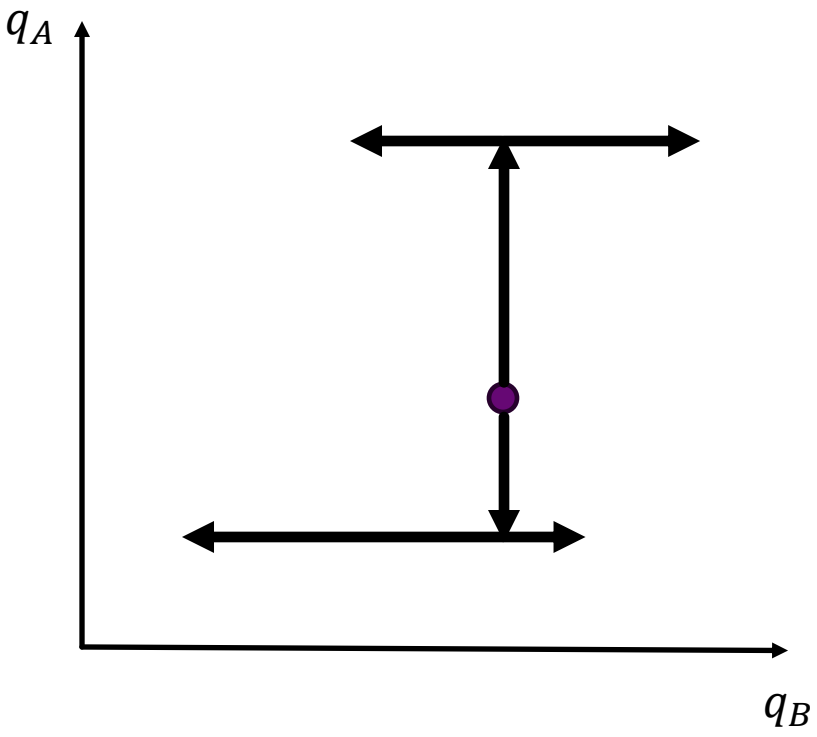


The 2 players update their believes according to the Joint probability distribution:

$\Pr(\theta_A, \theta_B, s)$	θ_{B1}	θ_{B2}	$\dots$	θ_{Bn}
θ_{A1}	$p_{11}(s)$	$p_{12}(s)$	$\dots$	$p_{1n}(s)$
θ_{A2}	$p_{21}(s)$	$p_{22}(s)$	$\dots$	$p_{2n}(s)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
θ_{Am}	$p_{m1}(s)$	$p_{m2}(s)$	$\dots$	$p_{mn}(s)$

Compare them!

Bayesian Conversation



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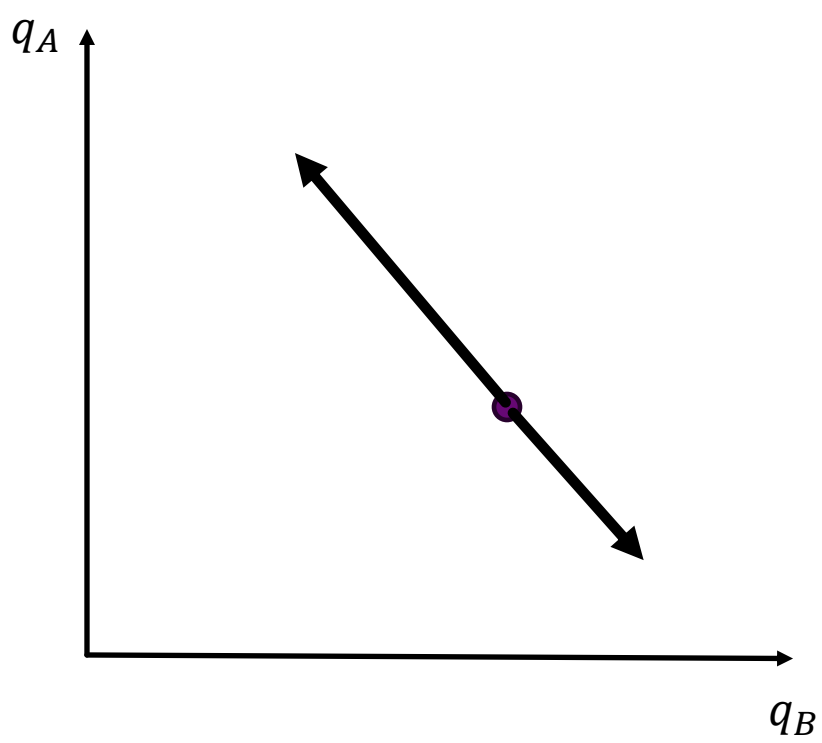
Mediator Protocol



Question: How much the players' expected utilities can be improved by this larger set of belief distributions?

Ex-ante IR – Equivalence

(Theorem 3.1) For any utility function, the range of expected utilities that can be implemented by ex-ante IR Bayesian conversations is equivalent to the range of expected utilities that can be implemented by ex-ante IR mediator protocols.

However, this property doesn't hold under ex-post IR (There exists a gap).

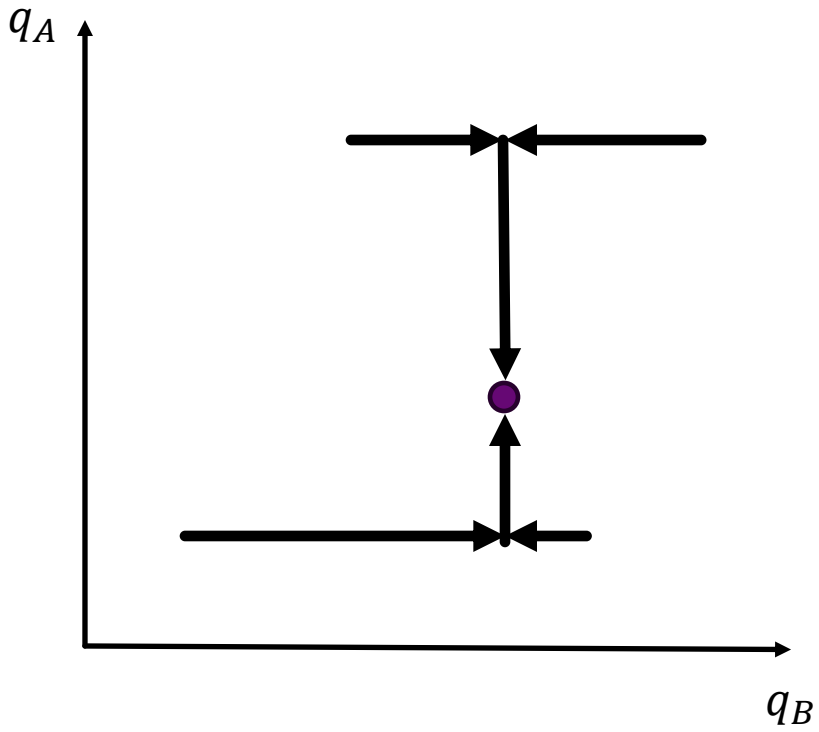
To investigate this we need to propose an algorithm to find the optimal ex-post IR Bayesian conversation.

Ex-post IR – Algorithm Design

Key Observation:

One-to-one correspondence between coordinates and utility

Dynamic Programming!



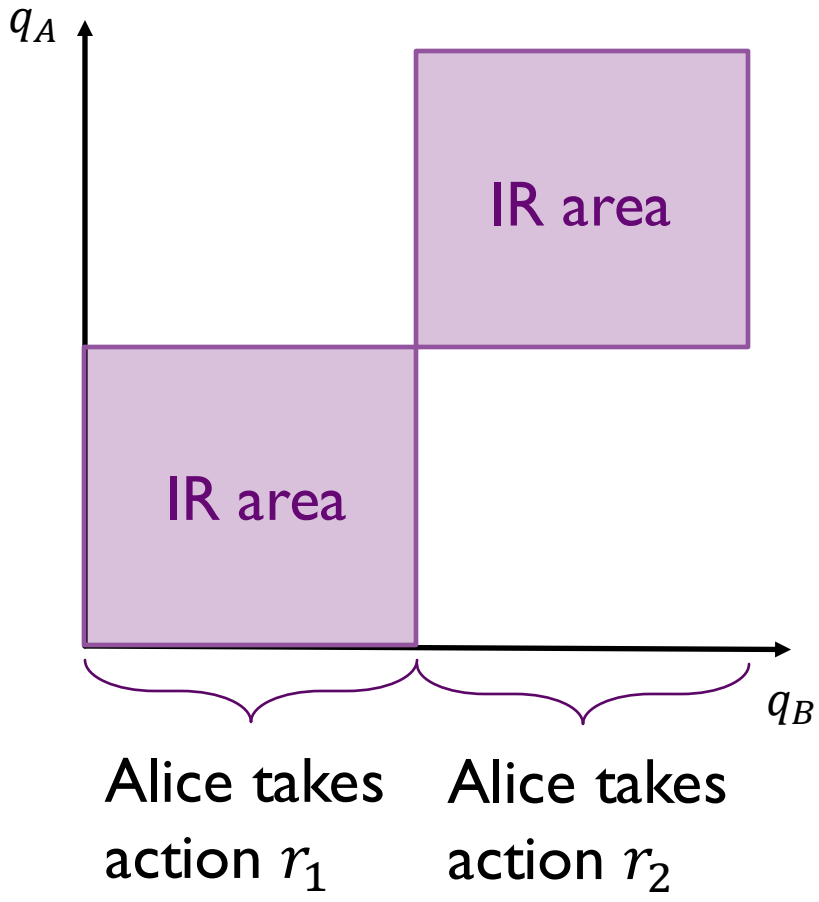
Ex-post IR – Exact Discretization

Key Observation:

- Alice's action locally remain the same
- Bob's IR notation satisfied within an interval

Smaller Rectangular,

within which utility is bilinear

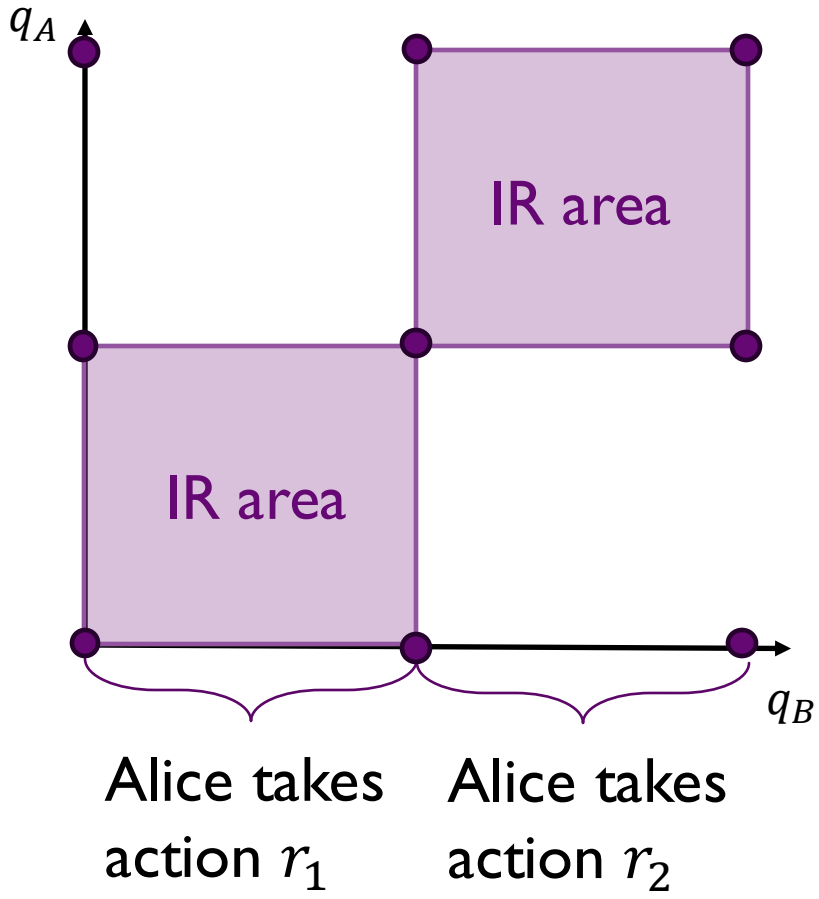


Ex-post IR – Exact Discretization

(Theorem 4.1)

The entire dynamic programming process ***only needs to act on the vertices*** of these small rectangles, achieving ***exact discretization***.

- Gap between BC and MP
- Exists a games need infinite round to converge



Thanks!